there is a structure in bias dependence of the coefficients for pure shear [$(A-B)$, $(C-E)$, and $(D-E)$] that does not appear in the coefficients for hydrostatic pressure. This structure is noteworthy because it occurs over a narrow bias range which is symmetric about the origin. All of the pure shear coefficients mentioned above increase in absolute magnitude by $20\pm4\%$ in going from reverse bias to forward bias.

At large negative biases ($V<-0.13$ volts) the stress coefficients show a shallow minimum. In this bias range one expects the onset of direct tunneling into the $k=(000)$ conduction-band states. Since compression increases the direct gap at (0,0,0), and hence the onset voltage for direct tunneling, a negative contribution to the stress coefficients is expected in this range. The ob-observed dip in the stress coefficients is less pronounced in As-doped junctions than in Sb-doped junctions because the magnitude of the impurity-assisted current is much more nearly equal to the direct tunneling current in this bias range.

IV. DISCUSSION AND CONCLUSIONS

The absence of a large positive shear contribution to the stress coefficient D in As-doped junctions shows that in this case the tunneling wave functions on the n-type side are not associated with individual (111) conduction-band valleys. This conclusion is not unexpected, since phonon assistance would be very likely for such a case regardless of the details of the tunneling process. Furthermore, the arsenic donors are known to produce a strong localized central cell potential[11] which introduces components into the wave function from large parts of the Brillouin zone including all of the valleys and the region around $k=0$. Since the electron-phonon interaction in germanium is not large, a small admixture of (000) character to the electron wave function should result in a predominant tunneling current which is not assisted by phonons.

At present, there is no theory of impurity-induced indirect tunneling with which the experimental results should be compared. Other measurements on this process have been compared[12] with some success with the expression for direct interband tunneling presented by Kane,[2] but the agreement found in these cases should not be regarded, in our opinion, as evidence that the tunneling process under consideration is the one analyzed by Kane.

However, the form of the exponential factor in the tunneling expression seems to be quite independent of the details of the tunneling process. Therefore, one might tentatively use Kane's expression for direct tunneling with a different interpretation of the effective masses and the band gap for the interpretation of the shear dependence observed in As-doped diodes. The tunneling exponent depends on the projection of the reduced effective-mass tensor in the direction of the electric field. Since the effective-mass tensors are expected to deform under the influence of shear, it is interesting to see if this effect can give rise to the observed differences between the hydrostatic and uniaxial stress coefficients, and if so, how large the changes of the effective masses have to be. The reciprocal reduced effective-mass tensor is defined as

$$\mathbf{m}_r^{-1}=\mathbf{m}_v^{-1}+\mathbf{m}_c^{-1}. \tag{4}$$

In the present case $\mathbf{m}_v^{-1}$ is the reciprocal effective-mass tensor of the light-hole band and $\mathbf{m}_c^{-1}$ is a reciprocal mass tensor for the electrons. Because we are dealing with impurity-induced tunneling it is not certain whether the conduction-band minimum at $k=0$ or an appropriate average of the (111) conduction-band valleys determines $\mathbf{m}_c^{-1}$.

Since the tunneling experiments cannot separate the contributions of the hole and the electron masses, we describe the stress-induced changes of the reduced reciprocal effective-mass tensor by a single fourth-rank deformation-potential tensor $\mathbf{Q}$. It was found experimentally that $B=E$ and $A=C$. These results are consistent with the assumption that $\mathbf{m}_r^{-1}$ is spherically symmetric at zero stress and that cubic symmetry holds for $\mathbf{Q}$. The fact that A and C are more negative than B and E indicates that the reduced effective mass is increased in the plane perpendicular to the [100] compression axis. A [110] compression will deform the reduced effective-mass sphere into a general ellipsoid. The positive shear contribution $(D-E)$ implies a reduction of the mass component along [1$\bar{1}$0] for shear resulting from a uniaxial compression along [110].

In the following the magnitudes of the deformation potentials Q_{11}, Q_{12}, and Q_{44} will be estimated. We assume that the tunnel current is given by an expression of the familiar form[2,13]

$$I=C\times D\exp(-\alpha)\,, \tag{5}$$

$$\alpha=\lambda E^{3/2}m_{rx}^{1/2}/hF\,. \tag{6}$$

Here, λ is a numerical constant of the order of unity that depends on the particular theory and on the way the average field is calculated but that need not be known for our argument. E is the relevant band gap and F is the average junction field which also does not need to be known.

If one assumes that (similar to the cases of direct and phonon-assisted tunneling[8]) the major effect of pressure results from the exponential factor in Eq. (5), then the coefficient for hydrostatic pressure is given by

$$B=-\frac{1}{3}\frac{d(\ln I)}{dp}=-\frac{\alpha}{3}\left[\frac{3}{2}\frac{d\ln E}{dp}+\frac{1}{2}\frac{d(\ln m_{rx})}{dp}-\frac{d\ln F}{dp}\right]. \tag{7}$$

[11] P. J. Price, Phys. Rev. **104**, 1223 (1956); W. Kohn, in *Solid-State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1957), Vol. 5; H. Fritzsche, Phys. Rev. **115**, 336 (1959).

[12] D. Meyerhofer, G. A. Brown, and H. S. Sommers, Phys. Rev. **126**, 1329 (1962).

[13] K. B. McAfee, E. J. Ryder, W. Shockley, and M. Sparks, Phys. Rev. **83**, 650 (1951).

With the approximate relations

$$\frac{d\ln m_{rx}}{dp} \approx \frac{d\ln E}{dp}, \tag{8}$$

$$\frac{d\ln F}{dp} \approx \frac{1}{2}\frac{d\ln E}{dp}, \tag{9}$$

we obtain approximately

$$B \approx \frac{\alpha}{2}\frac{d\ln E}{dp}. \tag{10}$$

The factor $\frac{1}{3}$ has been included in the definitions of the hydrostatic pressure coefficients B and E [see Eq. (7)] because a uniaxial stress X produces a dilatation effect equivalent to a hydrostatic pressure of magnitude $p=X/3$. One therefore obtains for the effect of the shear part of the uniaxial stress

$$A-B = -\tfrac{1}{2}\alpha\left[\frac{d\ln m_{rx}}{dX(100)} - \frac{1}{3}\frac{d\ln m_{rx}}{dp}\right]. \tag{11}$$

The combination of Eqs. (10) and (11) yields the effect of pure shear on the reduced effective mass in the tunneling direction in terms of the measured stress coefficients and the effect of pressure on the band gap as

$$\frac{d\ln m_{rx}}{dX(100)} - \frac{1}{3}\frac{d\ln m_{rx}}{dp} = \frac{A-B}{B}\frac{d\ln E}{dp}, \tag{12}$$

with equivalent equations for the other orientations. The components of the deformation-potential tensor **Q** in the coordinate system of the cube axes are then

$$Q_{11} = -\left[\tfrac{1}{3} - \frac{2(A-B)}{B}\right]\frac{d\ln E}{dp},$$

$$Q_{12} = -\left[\tfrac{1}{3} + \frac{(A-B)}{B}\right]\frac{d\ln E}{dp}, \tag{13}$$

$$Q_{44} = \left[\frac{2(D-E)}{E} + \frac{(A-B)}{B}\right]\frac{d\ln E}{dp}.$$

If we assume the appropriate band gap for impurity-induced tunneling is the indirect gap, then $d\ln E/dp = 6.75\times10^{-12}$ cm²/dyn and the components of the deformation-potential tensor are, using the stress coefficients of the positive bias range,

$$Q_{11} = 4.5\times10^{-12}\ \text{cm}^2/\text{dyn},$$

$$Q_{12} = -5.6\times10^{-12}\ \text{cm}^2/\text{dyn},$$

$$Q_{44} = -4.4\times10^{-12}\ \text{cm}^2/\text{dyn}.$$

The deformation potentials of the reduced-mass tensor and of the appropriate tunneling band gap are found to be of the same order of magnitude. It is still uncertain, however, to which electron masses one refers by writing Eq. (4). The data on Sb-doped tunnel diodes[8] in the bias region where phonon-assisted tunneling dominates yield $(C-E)/E$ and hence a shear-induced mass change of the same order of magnitude as found here. In the bias range of direct interband tunneling those diodes showed a considerably smaller shear effect. This seems to indicate that the shear dependence on the impurity-induced tunneling of the As-doped diodes is not due to a change of the reduced mass of the direct gap at the zone center.

If the shear dependence of the As-doped diodes arises from a reduced mass involving the (111) conduction-band valleys, then the cubic symmetry shown by the data implies that the tunneling electron would have to be equally shared by all four valleys. Although this possibility is not in conflict with the observation[14] of considerable intervalley scattering in As-doped germanium, it is not included in the present theoretical formulations of tunneling.

The data also possess one other feature which remains unexplained. There is found to be some fine structure in the bias dependence of the shear part of the stress coefficient in the region around zero bias. The shear coefficients in the forward bias region are approximately 20% larger in magnitude compared to those of the reverse bias direction.

[14] W. P. Mason and T. B. Bateman, Phys. Rev. 134, A1387 (1964); P. J. Price and R. L. Hartman, J. Phys. Chem. Solids 25, 567 (1964).